DESIGN AND ANALYSIS OF THE PROPERTIES OF THE DELTA INVERSE ROBOT

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Introduction

Parallel manipulators have been studied in numerous publications [1-6], hence it is impossible to go over them exhaustively. But the manipulator Delta is one of the best known and most popular due to its original architecture enabling the orientation of its moving platform to remain a constant. It is known that each kinematic chain of the manipulator Delta consists of an input link connected to the base by means of a rotating joint, and of a parallelogram connected to a moving platform. Obviously, the architecture of the parallelogram is more complicated than the architecture of the input link, therefore the weight of the parallelogram may be greater than that of the input link. In our opinion, the parallelogram is to be situated closer to the base, in order to decrease the amplitude of its displacements and velocities. Hence, it could decrease the power of the actuators.

This article focuses on a new manipulator analogous to the manipulator Delta in which the parallelogram or the link containing two universal joints is connected to the base. The constraints imposed by the kinematic chains are considered. This article partially uses the recently obtained results [6]. The working volume is one the most important properties of manipulators. Here the working volumes of new manipulator sand of known manipulators are compared by using known algorithms [1]. The singularities have an influence on the working volume that is why the special configurations of both manipulators are considered by using the approaches based on the Jacobian matrices and on the theory of screws [1, 2, 4, 5].

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Description and kinematics of the structure

Let us examine the manipulator of which the structure is described at Fig. 1. This manipulator is composed of three identical legs (Fig. 2) linked to both the fixed base and the moving platform. Each leg is composed of one actuated revolute joint Q_i (i = 1, 2, 3) mounted onto the base and linked to an articulated parallelogram $A_iB_iC_iD_i$. Each parallelogram is connected to a rod T_iP_i through a revolute joint. Finally, rods T_iP_i are connected to the moving platform by means of another revolute joint P_i. Such a structure corresponds to a Delta robot of which the legs are turned. Therefore, the degrees of freedom of the manipulator are 3 translations of the platform.

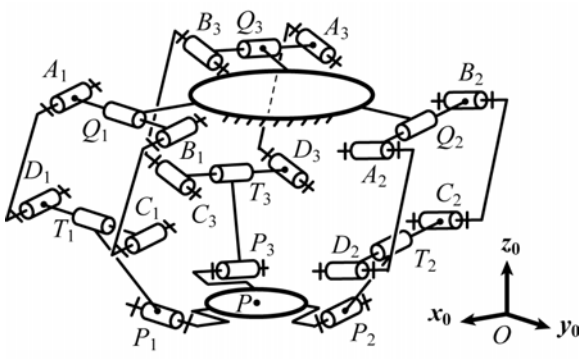

Fig. 1. Schematic representation of the Delta inverse robot.

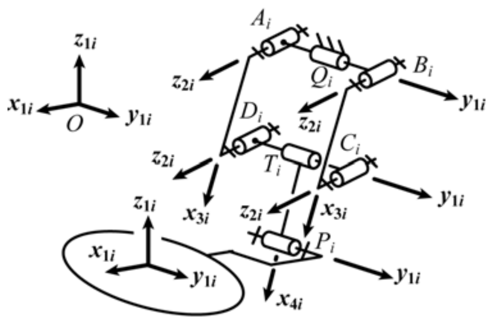

Fig. 2. Schema of one leg of the Delta inverse robot.

It is obvious that some pairs of the structures can be changed without losing the kinematic characteristics of the moving platform. For example, the parallelograms can be replaced by one rod linked to two cardan joints.

For our analysis, let us consider that triangle $Q_1Q_2Q_3$ is equilateral. Point O of the base frame $\boldsymbol{x_0y_0z_0}$ is located at the centre of the circumscribed circle to $Q_1Q_2Q_3$, $\boldsymbol{x_0}$ is directed along $\mathbf{OQ_1}$ and $\boldsymbol{z_0}$ is normal to the plane $Q_1Q_2Q_3$. We also consider that triangle $P_1P_2P_3$ is equilateral and that point P of the moving platform is located at the centre of the circumscribed circle to $P_1P_2P_3$. Point P is the controlled point with coordinate $[x, y, z]^T$ in the global frame.

Vectors $\boldsymbol{x_{1i}}$ correspond to the directions of lines OQ_i and $\boldsymbol{y_{1i}}$ to the directions of the revolute joints Q_i (Fig. 2) and they can be obtained with respect to the global frame after a rotation of angle γ_i around $\boldsymbol{z_0}$ axis (γ_i = {0°, 120°, 240°}). Vector $\boldsymbol{z_{2i}}$ can be obtained from frame (O, $\boldsymbol{x_{1i}}$, $\boldsymbol{y_{1i}}$, $\boldsymbol{z_{1i}}$) after a rotation of angle θ_i around $\boldsymbol{y_{1i}}$ axis, vector $\boldsymbol{x_{3i}}$ from frame (O, $\boldsymbol{x_{2i}}$, $\boldsymbol{y_{2i}}$, $\boldsymbol{z_{2i}}$) after a rotation of angle α_i around $\boldsymbol{z_{2i}}$ axis and vector $\boldsymbol{x_{4i}}$ from frame (O, $\boldsymbol{x_{1i}}$, $\boldsymbol{y_{1i}}$, $\boldsymbol{z_{1i}}$) after a rotation of angle β_i around $\boldsymbol{y_{1i}}$ axis.

The expression of the vector $\mathbf{q} = [\theta_1,\ \theta_2,\ \theta_3]$ of input coordinates can be obtained from the closure equation:

$$\mathbf{OP}_i = \mathbf{OQ}_i + \mathbf{Q}_i\mathbf{T}_i + \mathbf{T}_i\mathbf{P}_i \qquad (1)$$

From expressions (1) after transformations, it can be deduced that:

$$x_{Qi} - x_{Pi} + L_p \cos\alpha_i \cos\theta_i = -L_b \cos\beta_i \qquad (2)$$

$$y_{Qi} - y_{Pi} + L_p \sin\alpha_i = 0 \qquad (3)$$

$$z_{Qi} - z_{Pi} - L_p \cos\alpha_i \sin\theta_i = L_b \sin\beta_i \qquad (4)$$

where:

- R_b and R_p are the radii of the circumscribed circles to $Q_1Q_2Q_3$ and respectively $P_1P_2P_3$;
- L_p is the length of the rods A_iD_i and B_iC_i;
- L_b is the length of the rods T_iP_i.

Raising to square both sides of equations (2) and (4), and summing, it can be found that:

$$f_i = \left(x_{Qi} - x_{Pi} + L_p \cos\alpha_i \cos\theta_i\right)^2 + \left(z_{Qi} - z_{Pi} - L_p \cos\alpha_i \sin\theta_i\right)^2 - L_b^2 = 0 \qquad (5)$$

where the expressions of angles α_i can be deduced from (3):

$$\alpha_i = \{\alpha_{i1},\ \alpha_{i2}\} = \left\{\sin^{-1}\left(\frac{y_{Pi} - y_{Qi}}{L_p}\right),\ \pi - \sin^{-1}\left(\frac{y_{Pi} - y_{Qi}}{L_p}\right)\right\}. \qquad (6)$$

Finally, the expression of angles θ_i can be obtained from equation (5).

Differentiating equation (5) with respect to both articular and Cartesian coordinates, one can obtain the following expression [4]:

$$\mathbf{A}\,\mathbf{v} + \mathbf{B}\,\dot{\mathbf{q}} = \mathbf{0}\ . \qquad (7)$$

with

$$\mathbf{A} = \left[\frac{\partial f_i}{\partial v_j}\right],\ \mathbf{B} = \left[\frac{\partial f_i}{\partial \theta_j}\right],\ \mathbf{v} = \left[\dot{x} \quad \dot{y} \quad \dot{z}\right]^T \text{ and } \dot{\mathbf{q}} = \left[\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3\right]^T, \qquad (8)$$

where v_j corresponds to the j-th coordinate of vector $\mathbf{v}$.

Therefore, the kinematic model can be found from (8):

$$\mathbf{v} = \mathbf{J}\,\dot{\mathbf{q}}\text{, where } \mathbf{J} = -\mathbf{A}^{-1}\mathbf{B}\,. \qquad (9)$$

Thus, from expression (9), it is possible to determine the Type 1 singularity loci [4] of the manipulator, which can be found by analyzing the degeneracy of matrix **B**. It can be shown that its determinant is equal to $b_{11}b_{22}b_{33}$, which means that matrix **B** degenerates if $b_{ii} = 0$, for $i = 1, 2, 3$.

Therefore, b_{ii} is equal to zero if and only if:

- $\alpha_i = \pm 90°$; in this case, which corresponds to a change of working mode, a displacement of the input angle θ_i does not induce a displacement of the platform along $\boldsymbol{y_{1i}}$ axis;
- the expression $\sin\theta_i\left(x_{Qi} - x_{Pi} + L_p \cos\alpha_i \cos\theta_i\right) + \cos\theta_i\left(z_{Qi} - z_{Pi} - L_p \cos\alpha_i \sin\theta_i\right)$ vanishes.

Introducing equations (2) and (4) into the following expression, we can find:

$$0 = -L_b \sin\theta_i \cos\beta_i + L_b \cos\theta_i \sin\beta_i\,, \qquad (10)$$

which can be written in the form

$$0 = L_b \sin(\theta_i - \beta_i)\,. \qquad (11)$$

Thus, a Type 1 singularity also exists if $\theta_i = \beta_i + n\pi$ ($n = 0, 1, 2$ ...). In such a case, a displacement of the input angle θ_i does not induce a displacement of the platform in the $O\boldsymbol{x_{1i}z_{1i}}$ plane;

Starting with equation (9), Type 2 singularities can be found [9, 10]. But for such an analysis, an augmented matrix should be used. Therefore, in order to investigate the Type 2 singularity it would be preferable to use another method which might more easily give simple results without complicated computation. Such an approach is used in the following section.

The analysis of constraints of Delta and Delta inverse robot via the theory of screws

The aim of this part is to find the Type 2 singularities of the Delta inverse robot (Fig. 3) using the theory of screw. One can find the wrenches reciprocal to the unit screws of the kinematic pairs. These wrenches for the Delta robot are determined in [6].

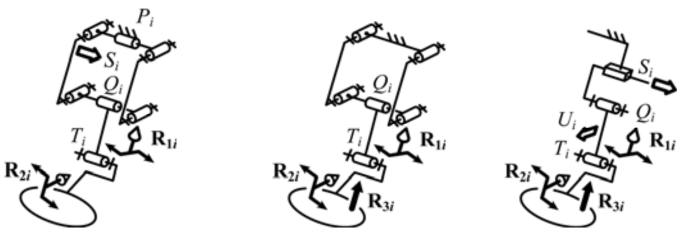

Fig. 3. Representation of one kinematic chain of the Delta inverse robot.

The axes of the rotating kinematic pairs P_i, Q_i, T_i are parallel. The parallelogram (Fig. 3) can be represented as a sliding kinematic pair S_i whose axis is perpendicular to the line P_iQ_i and parallel to the infinite small movement of the output link of the parallelogram. Each kinematic chain determines two wrenches $\mathbf{R}_{1i}$ $(0, 0, 0, r^{o}_{1ix}, r^{o}_{1iy}, r^{o}_{1iz})$ and $\mathbf{R}_{2i}$ $(0, 0, 0, r^{o}_{2ix}, r^{o}_{2iy}, r^{o}_{2iz})$ (i = 1, 2, 3) of infinite pitch (moments) that are reciprocal to the unit screws of the axes of the kinematic pairs. Both of them are perpendicular to the axes of the pairs P_i, Q_i, T_i. The system of constraint wrenches determining the degree of freedom of the platform has Plücker coordinates:

$$\begin{bmatrix} 0 & 0 & 0 & r^0_{11x} & r^0_{11y} & r^0_{11z} \\ 0 & 0 & 0 & r^0_{21x} & r^0_{21y} & r^0_{21z} \\ 0 & 0 & 0 & r^0_{12x} & r^0_{12y} & r^0_{12z} \\ 0 & 0 & 0 & r^0_{22x} & r^0_{22y} & r^0_{22z} \\ 0 & 0 & 0 & r^0_{13x} & r^0_{13y} & r^0_{13z} \\ 0 & 0 & 0 & r^0_{23x} & r^0_{23y} & r^0_{23z} \end{bmatrix} \tag{12}$$

All these six wrenches are moments, the rank of the matrix is equal to three.

When the driving pairs P_i are fixed, we have nine wrenches reciprocal to the unit screws of the axes of the passive kinematic pairs. Three additional wrenches are of zero pitch, their axes are perpendicular to the axes S_i of “sliding pairs” and to the vectors $\boldsymbol{U}_i$ perpendicular to the axes of the pairs Q_i and T_i and to the lines Q_iT_i.

As the system presents nine wrenches, it corresponds with statically undetermined structure. The criterion of singularity is the rank of the matrix. If it is equal to five or less then the mechanism is in singular configuration. For example a singular configuration exists when the lines Q_iT_i (i = 1, 2, 3) of all connecting chains are coplanar or parallel.

Other structure of the Delta robot in which the mobile and immobile platforms changed the places (Fig. 4) exists. The corresponding kinematic chain consists of two universal joints P_i and Q_i and one turning kinematic pair T_i. Each kinematic chain determines one wrench of infinite pitch (moment) $\mathbf{R}_{1i}$ $(0, 0, 0, r^{o}_{1ix}, r^{o}_{1iy}, r^{o}_{1iz})$ which is reciprocal to the unit screws of the axes of the kinematic. It is perpendicular to the axes of all turning kinematic pairs.

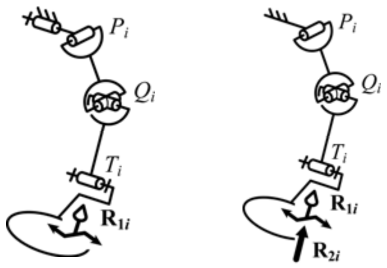

Fig. 4. Other possible kinematic chain of the Delta inverse robot with cardan joints.

The wrenches reciprocal to the unit screws of the passive kinematic pairs have Plücker coordinates:

$$\begin{bmatrix} 0 & 0 & 0 & r^0_{11x} & r^0_{11y} & r^0_{11z} \\ r_{21x} & r_{21y} & r_{21z} & r^0_{21x} & r^0_{21y} & r^0_{21z} \\ 0 & 0 & 0 & r^0_{12x} & r^0_{12y} & r^0_{12z} \\ r_{22x} & r_{22y} & r_{22z} & r^0_{22x} & r^0_{22y} & r^0_{22z} \\ 0 & 0 & 0 & r^0_{13x} & r^0_{13y} & r^0_{13z} \\ r_{23x} & r_{23y} & r_{23z} & r^0_{23x} & r^0_{23y} & r^0_{23z} \end{bmatrix} \tag{13}$$

Here the mentioned wrenches $\mathbf{R}_{1i}$ (0, 0, 0, r^o_{1ix}, r^o_{1iy}, r^o_{1iz}) are of infinite pitch and $\mathbf{R}_{2i}$ (r_{2ix}, r_{2iy}, r_{2iz}, r^o_{2ix}, r^o_{2iy}, r^o_{2iz}) are of the zero pitch (vectors) (i = 1, 2, 3). The axes of the wrenches $\mathbf{R}_{2i}$ intersect the points Q_i as well as the axes of all passive kinematic pairs of the corresponding kinematic chains.

The criterion of singularity is the rank of the matrix (13). If it is equal to five or less then the mechanism is in singular configuration. Similarly to previous case a singular configuration exists when the lines Q_iT_i (i = 1, 2, 3) of all connecting kinematic chains are coplanar or parallel.

The following part concerns the comparison of the size of the workspaces of both Delta and Delta inverse manipulators.

Workspace analysis

In this part, we would like to analyze the size of the workspace of both Delta and Delta inverse robots taking into account geometric limitations. The procedure used to compute the workspace of the manipulators taking into account geometric limitations is very simple. Firstly, we discretize the Cartesian space into n points (the discretization step is fixed to 0.02 m). For a point Q to belong to the workspace of the desired manipulator, it should comply with the geometric limitations, namely:

- values of angle θ_i computed using the inverse geometric models have to be comprised between 0° and 90°;
- for the Delta inverse robot, values of angle α_i computed using the inverse geometric models comprised of between -90° and 90° in order to avoid interferences with the base;
- the altitude of the platform has not to be superior to z = -0.3 m in order to avoid interferences with the base.

Thus, the workspaces of both manipulators can be computed. The results are presented in the figure 5. For these two robots, the following parameters are used, which correspond to the parameters of the SurgiScope® developed by company ISIS: R_b = 0.25 m, R_p = 0.2 m, L_p = 0.95 m, L_b = 0.75 m. It can be seen that, taking into account only the geometric limitations of the manipulators, the workspace of the Delta inverse robot (1.81 m^3) is larger than the workspace of the Delta robot (1.48 m^3).

(a) Delta inverse (b) Delta

Fig. 5. Workspaces of the manipulators under study taking into account the geometric limitations.

Conclusion

In this article, a new manipulator analogous to the manipulator Delta, in which the parallelogram or the link containing two universal joints is connected with the base, is presented. The working volumes of the new manipulator and of the known manipulator with analogous links are compared. The working volume of the new manipulator is greater than the working volume of the known Delta manipulator; therefore the applications of the new manipulator are more numerous.

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SUMMARY

The article focuses on a new manipulator whose prototype is similar to the well known Delta robot. However, in the new manipulator, the link comprising a parallelogram or two universal joints is located on the base. In our simulations, the links of the new mechanism have the same parameters that in the existing Delta robot, used in the SurgiScope® device commercialized by the company ISIS. It is shown that the working volume of the new manipulator is larger than the working volume of the existing manipulator.